6.5 - Properties of Logarithms

Warmup

1. $\log_{10} 7 + \log_{10}(n-2) = \log_{10} 6n$ 14

2. $\log_{10}(m+3) - \log_{10} m = \log_{10} 4$ 1

3. $\log_{10} x + \log_{10} x + \log_{10} x = \log_{10} 27$ 3

4. $4\log_5 x - \log_5 4 = \log_5 4$ 2

5. $\log_2 15 + \log_2 14 - \log_2 105 = \log_2 x$ 2

6. $2\log_3 x + \log_3 \frac{1}{10} = \log_3 5 + \log_3 2$ 10

7. $\log_4(x+2) + \log_4(x-4) = 2$ 6

6.5 - Properties of Logarithms

Properties

$$\log_3 x^5 = 5\log_3 x$$

$$\log_3 xy = \log_3 x + \log_3 y$$

$$\log_7 \frac{x}{y} = \log_7 x - \log_7 y$$

$$4\log_5 x - \log_5 4 = \log_5 x^4 - \log_5 4$$

$$= \log_5 \frac{x^4}{4}$$

Combine into one log

1. $2\log x^2 - 3\log\sqrt{x} - 2$

$$\log \frac{x^2\sqrt{x}}{100}$$

2. $\log_4\left(x\left(\log_2 \frac{\sqrt{2}}{2}\right)\right) - \log_8 16$

$$\log_4 \frac{-x\sqrt[3]{2}}{16}$$

6.6 - Solving Exponential and Logarithmic Equations

Solve

$$3^{\log_3 x} = 3x - 4$$

$$x = 3x - 4$$

$$x = 2$$

Simplify

1. $6^{\log_6(x+2)} = 2x - 6$

$x = 8$

2. $4^{\log_2(x+1)} = 25$

$x = 4, \cancel{-6}$

6.5 - Properties of Logarithms

Solve

$$10^x = 27$$

$$\log_{10} 10^x = \log_{10} 27$$

$x = \log_{10} 27$ use a calculator

$$x = 1.43...$$

6.5 - Properties of Logarithms

Solve

$$10^x = 27$$

$$\log_{10} 10^x = \log_{10} 27$$

$x = \log_{10} 27$ use a calculator

$$x = 1.43...$$

Practice

1. $10^{-3x} = 0.28$

0.18

2. $100^{-x} = \log_3(3 \log_5 125)$

-0.15

What about if not the same base?

$$5^x = 27$$

6.5 - Properties of Logarithms

Logarithm Change of Base Property Proof

$$k = \log_a x$$

$$a^k = x$$

$$\log_b(a^k) = \log_b x$$

$$k \log_b a = \log_b x$$

$$k = \frac{\log_b x}{\log_b a}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$5^k = 27$$

$$k = \log_5 27$$

$$k = \frac{\log 27}{\log 5}$$

$$k = \frac{1.431}{0.699}$$

$$k = 2.048$$

$$5^{2.048} = 27$$

6.5 - Properties of Logarithms

Changing Bases

$$5^k = 27$$

$$k = \log_5 27$$

$$k = \frac{\log 27}{\log 5} = 2.048$$

Simplify

1. $3^x = 40$

$x = 3.36$

2. $12^{-2x} = \log_2 133$

$x = -0.39$

What about?

$$6^{(x+2)} = 24^{(-2x+1)}$$

6.6 - Solving Exponential and Logarithmic Equations

Solve the expression

$2e^{12x} = 17 \quad e^{12x} = \frac{17}{2} \quad 12x = \log_e 8.5 \quad x = 0.17833$

Practice

1) $\frac{50}{1 + e^{-x}} = 4$

$x = -2.44$

2) $e^{2x} - 3e^{x} + 2 = 0$

$x = 0, \ln 2$

6.6 - Solving Exponential and Logarithmic Equations

Solve the expression

$\log x + \log(x-1) = \log(4x)$

$\log x(x-1) = \log(4x)$

$\log(x^2 - x) = \log(4x)$

$x^2 - x = 4x$

$x^2 - 5x = 0$

$x(x-5) = 0$

$x = \cancel{0}, 5$

Practice

1) $\log_2 3 + \log_2 x = \log_2 5 + \log_2(x-2)$

$x = 5$

2) $\log_2(x^2 - x - 2) = \log_2 2$

$x = -2, 3$

6.6 - Solving Exponential and Logarithmic Equations

Solving Equations

$$12^{2x} = 48$$

$$\log_{12} 12^{2x} = \log_{12} 48$$

$$2x = \log_{12} 48$$

$$x = \frac{\log 48}{2 \log 12} = 0.8$$

$$\log_e x = \ln x$$

$$\ln e^5 = 5$$

$$\ln 4 - \ln 10 = \ln \frac{4}{10}$$

$$= \ln 0.4 = -0.92$$

Practice

1. $\ln 3x = 2$

$x = 2.46$

2. $\ln 3 + \ln 2x = \ln 36$

$x = 6$

3. $3e^{2x} + 2 = 50$

$x = 1.39$